Capítulo 8

Calcular el valor esperado de $<\phi_a\phi_b\phi_c\phi_d>$

$$S[\phi] = \frac{m^2}{2}\phi^T A\phi \quad (1)$$

$$Z[J] = \int \mathcal{D}\phi\ e^{-S[\phi]+\phi^T J} = e^{\frac{1}{2m^2}J^T A^{-1} J} \cdot \frac{\left(\sqrt{2\pi}\right)^n}{m^n\sqrt{\det A}} \quad (2)$$

$$Z[0] = \frac{\left(\sqrt{2\pi}\right)^n}{m^n\sqrt{\det A}} \quad (3)$$

El factor anterior al hacer las derivadas y sustituir para J=0 se va a simplificar, al aparecer en el numerador y denominador de la fórmula siguiente:

$$<\phi_a\phi_b\phi_c\phi_d> = \frac{1}{Z[0]}\left[\frac{\partial}{\partial_{Ja}}\frac{\partial}{\partial_{Jb}}\frac{\partial}{\partial_{Jc}}\frac{\partial}{\partial_{Jd}}Z[J]\right]_{J=0} = \left[\frac{\partial}{\partial_{Ja}}\frac{\partial}{\partial_{Jb}}\frac{\partial}{\partial_{Jc}}\frac{\partial}{\partial_{Jd}}e^{\frac{1}{2m^2}J^T A^{-1} J}\right]_{J=0} \quad (4)$$

$Hacemos\ \frac{1}{2m^2}J^T A^{-1} J = a_{ij}x^i x^j\ y\ \frac{\partial}{\partial_{Jn}} = \delta_n$ para simplificar los cálculos

$$\delta_d e^{a_{ij}x^i x^j} = e^{a_{ij}x^i x^j}\cdot\delta_d(a_{ij}x^i x^j) = e^{a_{ij}x^i x^j}\cdot(a_{ij}(\delta_d x^i)x^j + a_{ij}x^i\delta_d x^j) = e^{a_{ij}x^i x^j}\cdot(a_{dj}x^j + a_{id}x^i)$$

$$\delta_c\left(e^{a_{ij}x^i x^j}\cdot\left(a_{dj}x^j + a_{id}x^i\right)\right) = \delta_c e^{a_{ij}x^i x^j}\cdot\left(a_{dj}x^j + a_{id}x^i\right) + e^{a_{ij}x^i x^j}\cdot\left(a_{dc} + a_{cd}\right) =$$
$$= e^{a_{ij}x^i x^j}\cdot\left(a_{cj}x^j + a_{ic}x^i\right)\cdot\left(a_{dj}x^j + a_{id}x^i\right) + e^{a_{ij}x^i x^j}\cdot 2a_{cd}$$

$$\delta_b\left(e^{a_{ij}x^i x^j}\cdot\left[\left(a_{cj}x^j + a_{ic}x^i\right)\cdot\left(a_{dj}x^j + a_{id}x^i\right) + 2a_{cd}\right]\right) =$$
$$= e^{a_{ij}x^i x^j}\cdot\left(a_{bj}x^j + a_{ib}x^i\right)\cdot\left[\left(a_{cj}x^j + a_{ic}x^i\right)\cdot\left(a_{dj}x^j + a_{id}x^i\right) + 2a_{cd}\right] +$$
$$+ e^{a_{ij}x^i x^j}\cdot\left[2a_{bc}\cdot\left(a_{dj}x^j + a_{id}x^i\right) + \left(a_{cj}x^j + a_{ic}x^i\right)\cdot 2a_{bd}\right] = exp.$$

$$\delta_a(exp.) = e^{a_{ij}x^i x^j}\cdot\left(a_{aj}x^j + a_{ia}x^i\right)\cdot\left(a_{bj}x^j + a_{ib}x^i\right)\cdot\left[\left(a_{cj}x^j + a_{ic}x^i\right)\cdot\left(a_{dj}x^j + a_{id}x^i\right) + 2a_{cd}\right] +$$
$$+ e^{a_{ij}x^i x^j}\cdot\left[2a_{ab}\cdot\left(\left(a_{cj}x^j + a_{ic}x^i\right)\cdot\left(a_{dj}x^j + a_{id}x^i\right) + 2a_{cd}\right) + \left(a_{bj}x^j + a_{ib}x^i\right)\cdot\left(2_{ac}\cdot\left(a_{dj}x^j + a_{id}x^i\right) +\right.\right.$$
$$\left.\left. + \left(a_{cj}x^j + a_{ic}x^i\right)\cdot 2a_{ad}\right)\right] + e^{a_{ij}x^i x^j}\cdot[\left(a_{aj}x^j + a_{ia}x^i\right)\cdot(2a_{bc}\cdot\left(a_{dj}x^j + a_{id}x^i\right) + \left(a_{cj}x^j + a_{ic}x^i\right)\cdot$$
$$2a_{bd}) +$$
$$+ 2a_{bc}\cdot 2a_{ad} + 2a_{ac}\cdot 2a_{bd})] = final$$

$$<\phi_a\phi_b\phi_c\phi_d> = [final]_{x=0} = 4a_{ab}a_{cd} + 4a_{bc}a_{ad} + 4a_{ac}a_{bd} = \frac{1}{m^4}\left(A^{-1}_{ab}A^{-1}_{cd} + A^{-1}_{bc}A^{-1}_{ad} + A^{-1}_{ac}A^{-1}_{bd}\right)$$